Workshop Young perspectives on irreducible holomorphic symplectic manifolds

Titles & abstracts

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Pietro Beri (Université Paris-Cité)
A birational involution on the Hilbert scheme cube of a K3 surface of genus 10
Birational automorphisms of Hilbert schemes of points on algebraic K3 surfaces of Picard rank one were recently classified, in a joint work with Al. Cattaneo. In particular, the existence of a (unique) birational involution of the Hilbert cube of a general K3 surface of genus 10 has been established. This kind of existence result does not give any hint about a geometric description of the involution; typically, finding such a description is a complex problem. In this talk, we will explicitly describe this involution, in terms of the Mukai model of the surface. This is a joint work with L. Manivel.

Valeria Bertini (Universidade do Porto)
Terminalization of quotients of hyperkähler manifolds via symplectic actions
To produce irreducible symplectic varieties, we can consider moduli spaces of sheaves on trivial canonical surfaces or partial resolution of symplectic quotients of smooth hyperkähler manifolds. In this talk, I will focus on the second class of examples, especially in the case of fourfolds. In order to produce new examples, I will start from the known hyperkähler fourfolds (Hilbert schemes and generalized Kummer) and act symplectically on them with automorphisms induced by the underlying surface, for which a systematic analysis is possible. This is the content of a work in progress with A. Capasso, O. Debarre, A. Grossi, M. Mauri and E. Mazzon.

Enrico Fatighenti (Università di Bologna)
Fano varieties of K3 type and their properties
Fano varieties of K3 type are a special class of Fano varieties, which are usually studied for their link with hyperkähler geometry, rationality properties, and much more. In this talk, we will recap some recent results, obtained jointly with M. Bernardara, L. Manivel, G. Mongardi, and F. Tanturri, that focus on the explicit construction of examples and the study of their Hodge-theoretical properties.

Grégoire Menet (Université de Lille)
Involutions on moduli spaces of stable sheaves on a K3 surface
Let \( M \) be a smooth moduli space of stable sheaves on a K3 surface; \( M \) is equivalent by deformation to a Hilbert scheme of points on a K3 surface. In the literature, there exist very few geometrical constructions of automorphisms on such manifolds. In this talk, we propose a method to construct involutions on \( M \) based on dualizing the evaluation map. The constructed involutions are non-natural (i.e. involutions which are not induced by an involution on a K3 surface). It is a joint work in progress with D. Faenzi and Y. Prieto Montañez.
Marc Nieper-Wißkirchen (Universität Augsburg)

*On the cohomology of the generalized Kummer varieties and topological Lefschetz numbers*

Generalized Kummer varieties give an example of an irreducible holomorphic-symplectic manifold in each allowed dimension. In the talk, we recall the construction of a generalized Kummer variety $K$ from its associated Hilbert Scheme of points on a torus $A$. We then describe how the cohomology of $K$ can be described in terms of the cohomology of $A$. Using this description, a formula for the topological Lefschetz numbers of natural automorphisms of $A$ will be derived. This formula is useful for studying the symmetries of a generalized Kummer variety.

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Claudio Onorati (Università degli Studi di Milano)

*Monodromy of singular symplectic moduli spaces*

I will report on a joint work in progress with A. Perego and A. Rapagnetta on the monodromy group of singular moduli spaces of sheaves on K3 surfaces. This extends previous results by E. Markman (smooth case) and myself (case in which there exists a desingularisation).

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Laura Pertusi (Università degli Studi di Milano)

*Moduli spaces of stable objects in Enriques categories*

Enriques categories are characterized by the property that their Serre functor is the composition of an involutive autoequivalence and the shift by 2. The bounded derived category of an Enriques surface is an example of Enriques category. Other interesting examples are provided by the Kuznetsov components of Gushel-Mukai threefolds and quartic double solids. In this talk, we study moduli spaces of semistable objects in the Kuznetsov components of Gushel-Mukai threefolds and quartic double solids with respect to Serre-invariant stability conditions. We provide a result of non-emptiness for these moduli spaces, by using the relation with certain moduli spaces on the associated K3 category. This is a joint work in preparation with A. Perry and X. Zhao.

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Yulieth Prieto Montañez (ICTP Trieste)

*Symplectic birational maps on projective IHS manifolds of $K3^{[n]}$ type*

We prove that projective IHS manifolds of $K3^{[n]}$ type admitting a non-trivial symplectic birational self-map of finite order are isomorphic to moduli spaces of stable (twisted) coherent sheaves on K3 surfaces. Motivated by this result, we analyze the case of birational involutions, inducing reflections on the movable cone, via birational geometry of the moduli spaces and Bridgeland stability conditions. This is a work in collaboration with D. Mattei and Y. Dutta.

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Lenny Taelman (Universiteit van Amsterdam)

*Derived equivalences between hyperkähler varieties*

The derived category of (coherent sheaves on) a smooth projective variety is a subtle invariant that is in general hard to ‘compute’, but that contains a lot of geometric information. It may happen that two non-isomorphic varieties have equivalent derived categories, in particular for varieties with trivial canonical bundle. Over the past 30 years, such phenomena have been extensively studied for abelian varieties and K3 surfaces, and are now rather well understood. Yet the case of higher-dimensional hyperkähler varieties remains rather mysterious. In this talk, I will survey a number of recent structural results on derived equivalences between higher-dimensional hyperkähler varieties, obtained by the speaker, T. Beckmann, E. Markman, and P. Magni.